GRADUATE COURSE ON CLUSTER ALGEBRAS

Math 5020 Topics in Algebra: Cluster Algebras (Fall 2018)
Instructor: Ralf Schiffler Office: Mont 336 Lectures: Mont 113, TT 12:30 – 1:45

Description: Cluster algebras are commutative algebras with a special combinatorial structure which are related to many different areas including Combinatorics, Representation Theory, Hyperbolic Geometry, Number Theory and Knot Theory. The subject is relatively young. The first paper on cluster algebras is published in 2002. Topics covered in the course include the following. Definition and examples, Laurent phenomenon and positivity, classifications, coefficients, relation to representation theory, interpretation in terms of triangulated surfaces, snake graphs and perfect matchings, continued fractions.

Homework: There will be no graded assignments, but students are encouraged to work on exercises given in class. These exercises are crucial to the understanding of the material.

Evaluation: Based on participation.

References

[1] Sergey Fomin, Lauren Williams and Andrei Zelevinsky, Introduction to Cluster Algebras (Chapters 1-3 Preliminary Version) https://arxiv.org/pdf/1608.05735.pdf 2
[2] Bernhard Keller, Cluster algebras, quiver representations and triangulated categories. (English summary) Triangulated categories, 76–160, London Math. Soc. Lecture Note Ser., 375, Cambridge Univ. Press, Cambridge, 2010
[3] Pierre-Guy Plamondon, Cluster Characters, CRM Short Courses, Homological Methods, Representation Theory, and Cluster Algebras, 2018 2
[4] Ralf Schiffler, Cluster algebras from surfaces CRM Short Courses, Homological Methods, Representation Theory, and Cluster Algebras, 2018. 2
[5] Ralf Schiffler, Cluster algebras and cluster categories, Lecture notes for the Latin American Algebra Colloquium, Sao Pedro, 2009. http://www.math.uconn.edu/~schiffler/saopedro.pdf 2
[6] Robert Marsh: Lecture notes on cluster algebras, Zurich Lecture Notes in Advanced Mathematics, 2013.
[7] Michael Gekhtman, Michael Shapiro and Alek Vainshtain: Cluster algebras and Poisson geometry, AMS Mathematical Surveys and Monographs, Volume 167, 2010.
[8] Ralf Schiffler: Quiver representations, CMS Books in Mathematics, 2014. 2

Tools:

Quiver mutation app https://webusers.imj-prg.fr/~bernhard.keller/quivermutation/ (by Bernhard Keller)

Cluster algebra portal http://www.math.lsa.umich.edu/~fomin/cluster.html (by Sergey Fomin)

Videos:

Master class at Arhus University, 23 lectures (by Sergey Fomin and Philippe DiFrancesco)

Cluster Algebras and Cluster Combinatorics, 4 Short courses at MSRI, 20 lectures (by Lauren Williams, Nathan Reading, Gregg Musiker and Ralf Schiffler)

Cluster algebras and categorification, 3 Lectures at CIRM (by Claire Amiot)

(1) Definition and examples
  (a) Quiver mutation, seed mutation
  (b) Recollections from algebra
  (c) Ground ring, ambient field, cluster algebra
  (d) Type $\mathbb{A}_n$
  (e) Grassmannian
  (f) Somos sequence
(2) Laurent phenomenon and positivity
  (a) Proof of LP [1]
(3) Classifications
  (a) Finite type
    (i) Dynkin diagrams, Coxeter groups
  (b) Finite mutation type
  (c) Acyclic type
  (d) Surface type
(4) Coefficients
  (a) trivial, principal, etc
  (b) F-polynomials, g-vectors, d-vectors, c-vectors
(5) Relation to Representation Theory
  (a) Quiver representations
    (i) chapters 1-3 from [8]
  (b) Cluster category [5, 3]
  (c) Cluster character [3]
(6) Surface Type [4]
  (a) Definition
  (b) Combinatorial formulas
  (c) Bases
(7) Snake graphs
  (a) Continued fractions
  (b) Jones polynomial
(8) Grassmannians